# PP44606

MIHÁLY BENCZE - ROMANIA

In all triangles $ABC$ holds:

$$\sum_{cyc} \frac{\cos A}{\sin^3 A} \geq \frac{3R^2}{rs}$$

*Solution by Daniel Sitaru, Claudia Nănuţi.*

$$\sum_{cyc} \frac{\cos A}{\sin^3 A} = \sum_{cyc} \frac{b^2+c^2-a^2}{2bc\sin A \cdot \sin^2 A} =$$

$$= \frac{1}{4F}\sum_{cyc} \frac{b^2+c^2-a^2}{\sin^2 A} = \frac{1}{4F}\sum_{cyc} \frac{b^2+c^2-a^2}{(\frac{a}{2R})^2} =$$

$$= \frac{4R^2}{4F}\sum_{cyc} \frac{b^2+c^2-a^2}{a^2} =$$

$$= \frac{R^2}{F}\left(\sum_{cyc} \frac{b^2}{a^2} + \sum_{cyc} \frac{c^2}{a^2} - 3\right) \overset{\text{AM-GM}}{\geq}$$

$$\geq \frac{R^2}{rs}\Big(3 \cdot \sqrt[3]{\frac{b^2}{a^2} \cdot \frac{c^2}{b^2} \cdot \frac{a^2}{c^2}} + 3\sqrt[3]{\frac{c^2}{a^2} \cdot \frac{a^2}{b^2} \cdot \frac{b^2}{c^2}} - 3\Big) =$$

$$= \frac{R^2}{rs}(3+3-3) = \frac{3R^2}{rs}$$

Equality holds for $a = b = c$.

□

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