PP44609

MIHÁLY BENCZE - ROMANIA

If $a, b > 0$ then:

$$\sqrt{a(a+2b)} + \sqrt{b(b+2a)} \geq 2\sqrt{3ab}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sqrt{a(a+2b)} + \sqrt{b(b+2a)} \overset{\text{AM-GM}}{\geq} 2\sqrt[4]{ab(a+2b)(b+2a)} \geq 2\sqrt{3ab}$$
$$\Leftrightarrow ab(a+2b)(b+2a) \geq 9a^2b^2$$
$$(a+2b)(b+2a) \geq 9ab$$
$$ab + 2a^2 + 2b^2 + 4ab - 9ab \geq 0$$
$$2a^2 + 2b^2 - 4ab \geq 0$$
$$a^2 - 2ab + b^2 \geq 0$$
$$(a-b)^2 \geq 0$$

Equality holds for $a = b$.

□

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