PP44634

MIHÁLY BENCZE - ROMANIA

If $0 < a \leq b$ then:

$$\int_a^b \frac{x^2+x+1}{(x+1)(2x+1)(x+2)}dx \geq \frac{1}{3}\ln\left(\frac{b+1}{a+1}\right)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

Lemma: If $x > 0$ then:

$$\frac{x^2+x+1}{(x+1)(x+2)(2x+1)} \geq \frac{1}{3(x+1)}$$

Proof.

$$3(x^2+x+1) \geq (x+2)(2x+1)$$
$$3x^2+3x+3 \geq 2x^2+x+4x+2$$
$$x^2-2x+1 \geq 0 \Leftrightarrow (x-1)^2 \geq 0$$

□

By lemma:

$$\int_a^b \frac{x^2+x+1}{(x+1)(2x+1)(x+2)}dx \geq \int_a^b \frac{1}{3(x+1)}dx =$$
$$= \frac{1}{3}\ln(x+1)\Big|_a^b = \frac{1}{3}(\ln(b+1)-\ln(a+1)) =$$
$$= \frac{1}{3}\ln\left(\frac{b+1}{a+1}\right)$$

Equality holds for $a = b$. □

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