PP44733

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{1}{c(a+b-c)} \geq \frac{3\sqrt{3}}{2sR}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc} \frac{1}{c(a+b-c)} \overset{\text{BERGSTRÖM}}{\geq} \frac{(1+1+1)^2}{\sum_{cyc} c(a+b-c)} =$$

$$= \frac{9}{2(ab+bc+ca)-(a^2+b^2+c^2)} = \frac{9}{2s^2+2r^2+8Rr-2s^2+2r^2+8Rr} =$$

$$= \frac{9}{4r^2+16Rr} \overset{\text{EULER}}{\geq} \frac{9}{4r \cdot \frac{R}{2}+16Rr} = \frac{9}{18Rr} =$$

$$= \frac{1}{2Rr} = \frac{3\sqrt{3}}{2R \cdot 3\sqrt{3}r} \overset{\text{MITRINOVIC}}{\geq} \frac{3\sqrt{3}}{2Rs}$$

Equality holds for $a = b = c$.

□

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