

PP44805

MIHÁLY BENCZE - ROMANIA

Compute:

$$\int_a^b \frac{(x^2 - (a+b)x + ab)^{2n+1}}{(x-a)^{2n+1} + (x-b)^{2n+1}} dx; n \in \mathbb{N}$$

Solution by Daniel Sitaru and Claudia Nănuți.

$$y = a + b - x; dx = -dy$$

$$x = a \Rightarrow y = b$$

$$x = b \Rightarrow y = a$$

$$\begin{aligned} \Omega &= \int_a^b \frac{(x-a)^{2n+1}(x-b)^{2n+1}}{(x-a)^{2n+1} + (x-b)^{2n+1}} dx = \\ &= - \int_b^a \frac{(a+b-y-a)^{2n+1}(a+b-y-b)^{2n+1}}{(a+b-y-a)^{2n+1} + (a+b-y-b)^{2n+1}} dy = \\ &= \int_a^b \frac{(b-y)^{2n+1}(a-y)^{2n+1}}{(b-y)^{2n+1} + (a-y)^{2n+1}} dy = \\ &= \int_a^b \frac{(-(y-b)^{2n+1}) \cdot (-(y-a)^{2n+1})}{-(y-b)^{2n+1} - (y-a)^{2n+1}} dy = \\ &= - \int_a^b \frac{(y-a)^{2n+1} \cdot (y-b)^{2n+1}}{(y-a)^{2n+1} + (y-b)^{2n+1}} dy = \\ &= - \int_a^b \frac{(x-a)^{2n+1} \cdot (x-b)^{2n+1}}{(x-a)^{2n+1} + (x-b)^{2n+1}} dx = -\Omega \\ &\quad \Omega = -\Omega \Rightarrow 2\Omega = 0 \Rightarrow \Omega = 0 \end{aligned}$$

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