PP44818

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$3\sqrt{\frac{3Rr}{2}} \leq s \leq \frac{4R+r}{\sqrt{3}}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$s \leq \frac{4R+r}{\sqrt{3}} \Leftrightarrow s\sqrt{3} \leq 4R+r \text{ is Doucet's inequality}$$

$$3\sqrt{\frac{3Rr}{2}} \leq s \Leftrightarrow s^2 \geq \frac{27Rr}{2} \text{ (to prove)}$$

$$s^2 \overset{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 \geq \frac{27Rr}{2} \Leftrightarrow$$

$$\Leftrightarrow 32Rr - 10r^2 \geq 27Rr$$

$$5Rr \geq 10r^2$$

$$R \geq 2r \text{ (Euler)}$$

Equality holds for $a = b = c$. □

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