# PP44833

MIHÁLY BENCZE - ROMANIA

In all triangles $ABC$ holds:

$$\frac{2R-r}{r} \leq \sum_{cyc} \frac{ab}{(b+c-a)(a-b+c)} \leq \frac{R^2-Rr+r^2}{r^2}$$

*Solution by Daniel Sitaru, Claudia Nănuţi.*

$$\sum_{cyc} \frac{ab}{(b+c-a)(a-b+c)} = \sum_{cyc} \frac{ab}{(2s-2a)(2s-2b)} =$$

$$= \frac{1}{4} \sum_{cyc} \frac{ab}{(s-a)(s-b)} = \frac{1}{4(s-a)(s-b)(s-c)} \sum_{cyc} ab(s-c) =$$

$$= \frac{s}{4s(s-a)(s-b)(s-c)} \left( \sum_{cyc} abs - 3abc \right) =$$

$$= \frac{s}{4F^2}(s \cdot (s^2+r^2+4Rr) - 12Rrs) =$$

$$= \frac{s^2}{4r^2s^2}(s^2+r^2+4Rr-12Rr) = \frac{s^2+r^2-8Rr}{4r^2}$$

Remains to prove that:

$$\frac{2R-r}{r} \leq \frac{s^2+r^2-8Rr}{4r^2} \leq \frac{R^2-Rr+r^2}{r^2}$$

$$8Rr - 4r^2 \leq s^2+r^2-8Rr \leq 4R^2-4Rr+4r^2$$

$$16Rr - 5r^2 \leq s^2 \leq 4R^2+4Rr+3r^2$$

which is Gerretsen's inequality.
Equality holds for $a = b = c$.

□

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