PP44834

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\frac{2(2R-r)}{R} \le \sum_{cyc} \frac{b+c-a}{a} \le \frac{2(R^2-Rr+r^2)}{Rr}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{b+c-a}{a} = \sum_{cyc} \frac{2s-2a}{a} = 2\sum_{cyc} \frac{s-a}{a} =$$

$$= 2\left(2\sum_{cyc} \frac{1}{a} - 3\right) = 2\left(s \cdot \frac{ab+bc+ca}{abc} - 3\right)$$

$$= 2\left(s \cdot \frac{s^2+r^2+4Rr}{4Rrs} - 3\right) = 2\left(\frac{s^2+r^2+4Rr}{4Rr} - 3\right) =$$

$$= 2 \cdot \frac{s^2+r^2+4Rr-12Rr}{4Rr} =$$

$$= \frac{s^2+r^2-8Rr}{2Rr}$$

Remains to prove that:

$$\frac{2(2R-r)}{R} \le \frac{s^2+r^2-8Rr}{2Rr} \le \frac{2R^2-2Rr+2r^2}{Rr}$$

$$8Rr-4r^2 \le s^2+r^2-8Rr \le 4R^2-4Rr+4r^2$$

$$16Rr-5r^2 \le s^2 \le 4R^2+4Rr+3r^2$$

which is Gerretsen's inequality.
Equality holds for $a=b=c$.

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