PP44843

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\frac{1}{(b+c-a)(a+c-b)}\geq\frac{1}{R^2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc}\frac{1}{(b+c-a)(a+c-b)}=\sum_{cyc}\frac{1}{(2s-2a)(2s-2b)}=$$

$$\frac{1}{4}\sum_{cyc}\frac{1}{(s-a)(s-b)}=\frac{1}{4(s-a)(s-b)(s-c)}\cdot\sum_{cyc}(s-c)=$$

$$=\frac{s}{4s(s-a)(s-b)(s-c)}\left(3s-\sum_{cyc}c\right)=$$

$$=\frac{s}{4F^2}(3s-2s)=\frac{s^2}{4r^2s^2}=\frac{1}{4r^2}=$$

$$=\frac{1}{(2r)^2}\overset{\text{EULER}}{\leq}\frac{1}{R^2}$$

Equality holds for $a=b=c$. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com