PP44846

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\prod_{cyc}\frac{r_a+r_b}{h_a+h_b}=\frac{4R^3}{r(s^2+r^2+2Rr)}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\prod_{cyc}\frac{r_a+r_b}{h_a+h_b}=\prod_{cyc}\frac{\frac{F}{s-a}+\frac{F}{s-b}}{\frac{2F}{a}+\frac{2F}{b}}=\frac{F}{8F}\prod_{cyc}\frac{\frac{s-a+s-b}{(s-a)(s-b)}}{\frac{a+b}{ab}}=$$

$$=\frac{1}{8}\prod_{cyc}\frac{c}{(s-a)(s-b)}\cdot\frac{ab}{a+b}=$$

$$=\frac{1}{8}\cdot\frac{(abc)^3}{(a+b)(b+c)(c+a)(s-a)^2(s-b)^2(s-c)^2}=$$

$$=\frac{(4RF)^3\cdot s^2}{8(a+b)(b+c)(c+a)\cdot s^2(s-a)^2(s-b)^2(s-c)^2}=$$

$$=\frac{64R^3F^3s^2}{8\cdot 2s(s^2+r^2+2Rr)\cdot F^4}=\frac{32R^3s^2}{8s(s^2+r^2+2Rr)\cdot F}$$

$$=\frac{32R^3s^2}{8s(s^2+r^2+2Rr)\cdot rs}=\frac{4R^3}{r(s^2+r^2+2Rr)}$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com