**PP44852**

MIHÁLY BENCZE - ROMANIA

In all acute $\Delta ABC$ holds:

$$\frac{6Rr-3r^2-2R^2}{2R^2} \le \cos A\cos B\cos C \le \frac{r^2}{2R^2}$$

*Solution by Daniel Sitaru and Claudia Nănuţi.*

$$\cos A\cos B\cos C \le \frac{r^2}{2R^2} \Leftrightarrow \frac{s^2-(2R+r)^2}{4R^2} \le \frac{r^2}{2R^2}$$

$$s^2-(2R+r)^2 \le 2r^2$$

$$s^2 \le 4R^2+4Rr+3r^2 \text{ (GERRETSEN)}$$

$$\cos A\cos B\cos C \ge \frac{6Rr-3r^2-2R^2}{2R^2} \Leftrightarrow$$

$$\Leftrightarrow \frac{s^2-(2R+r)^2}{4R^2} \ge \frac{6Rr-3r^2-2R^2}{2R^2} \Leftrightarrow$$

$$s^2 \ge (2R+r)^2+2(6Rr-3r^2-2R^2)$$

$$s^2 \ge 4R^2+4Rr+r^2+12Rr-6r^2-4R^2$$

$$s^2 \ge 16Rr-5r^2 \text{ (GERRETSEN)}$$

□

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