PP44860

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In all triangles ABC holds:

$$8r^2 - R^2 \leq \frac{abc}{a+b+c} \leq \frac{4s^2}{27}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\frac{abc}{a+b+c} \leq \frac{4s^2}{27} \Leftrightarrow \frac{4Rrs}{2s} \leq \frac{4s^2}{27} \Leftrightarrow$$

$$\Leftrightarrow 2Rr \leq \frac{4s^2}{27} \Leftrightarrow 54Rr \leq 4s^2 \Leftrightarrow s^2 \geq \frac{27Rr}{2} \text{ (to prove)}$$

$$s^2 \overset{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 \geq \frac{27Rr}{2}$$

$$32Rr - 10r^2 \geq 27Rr$$

$$5Rr \geq 10r^2$$

$$R \geq 2r \text{ (Euler)}$$

$$8r^2 - R^2 \leq \frac{abc}{a+b+c}$$

$$8r^2 - R^2 \leq \frac{4Rrs}{2s} \Leftrightarrow 8r^2 - R^2 \leq 2Rr \Leftrightarrow$$

$$\Leftrightarrow R^2 + 2Rr - 8r^2 \geq 0$$

$$R^2 - 2Rr + 4Rr - 8r^2 \geq 0$$

$$R(R-2r) + 4r(R-2r) \geq 0$$

$$(R-2r)(R+4r) \geq 0$$

$$R - 2r \geq 0$$

$$R \geq 2r \text{ (Euler)}$$

Equality holds for $a = b = c$. □

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