PP44861

MIHÁLY BENCZE - ROMANIA

If $a > 1; b \in (-2, 2)$ then compute:

$$\int_{\frac{1}{a}}^{a} \frac{\arctan x}{x^2 + bx + 1} dx$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\text{Let be } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow dx = -\frac{1}{y^2} dy$$

$$x = \frac{1}{a} \Rightarrow y = a$$

$$x = a \Rightarrow y = \frac{1}{a}$$

$$\Omega = \int_{\frac{1}{a}}^{a} \frac{\arctan x}{x^2 + bx + 1} dx = \int_{a}^{\frac{1}{a}} \frac{\arctan \frac{1}{y}}{\frac{1}{y^2} + \frac{b}{y} + 1} \cdot \left(\frac{-1}{y^2}\right) dy =$$

$$= -\int_{a}^{\frac{1}{a}} \frac{\arctan \frac{1}{y}}{1 + by + y^2} dy = \int_{a}^{\frac{1}{a}} \frac{\arctan \frac{1}{y}}{y^2 + by + 1} dy$$

$$\Omega = \int_{\frac{1}{a}}^{a} \frac{\arctan x}{x^2 + bx + 1} dx \tag{1}$$

$$\Omega = \int_{\frac{1}{a}}^{a} \frac{\arctan \frac{1}{x}}{x^2 + bx + 1} dx \tag{2}$$

By adding $(1); (2)$:

$$2\Omega = \int_{\frac{1}{a}}^{a} \frac{\arctan x + \arctan \frac{1}{x}}{x^2 + bx + 1} dx$$

$$2\Omega = \frac{\pi}{2} \int_{\frac{1}{a}}^{a} \frac{1}{(x + \frac{b}{2})^2 + \frac{3b^2}{4}} dx$$

$$\Omega = \frac{\pi}{4} \int_{\frac{1}{a}}^{a} \frac{(x + \frac{b}{2})'}{(x + \frac{b}{2})^2 + \frac{3b^2}{4}} dx$$

$$\Omega = \frac{\pi}{4} \cdot \frac{1}{\frac{b\sqrt{3}}{2}} \arctan\left(\frac{x + \frac{b}{2}}{\frac{b\sqrt{3}}{2}}\right)\Bigg|_{\frac{1}{a}}^{a}$$

$$\Omega = \frac{\pi}{2\sqrt{3}b}\left(\arctan\left(\frac{2a + b}{b\sqrt{3}}\right) - \arctan\left(\frac{2 + ab}{b\sqrt{3}}\right)\right)$$

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