PP44905

MIHÁLY BENCZE - ROMANIA

Find all $x, y, z \in \mathbb{R}$ for which:

$$16^x + 16^y + 16^z \le 2^{x+y+z}(2^x + 2^y + 2^z)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$2 \cdot 16^x + 16^y + 16^z \overset{\text{AM-GM}}{\ge} 4 \cdot \sqrt[4]{2^{4x} \cdot 2^{4x} \cdot 2^{4y} \cdot 2^{4z}} =$$
$$= 4 \cdot \sqrt[4]{(2^x)^4 \cdot (2^x)^4 \cdot (2^y)^4 \cdot (2^z)^4} = 4 \cdot 2^{2x+y+z}$$

$$2 \cdot 16^x + 16^y + 16^z \ge 4 \cdot 2^{2x+y+z} \tag{1}$$

Analogous:

$$16^x + 2 \cdot 16^y + 16^z \ge 4 \cdot 2^{x+2y+z} \tag{2}$$

$$16^x + 16^y + 2 \cdot 16^z \ge 4 \cdot 2^{x+y+2z} \tag{3}$$

By adding $(1); (2); (3)$:

$$4(16^x + 16^y + 16^z) \ge 4 \cdot 2^{x+y+z}(2^x + 2^y + 2^z)$$

$$16^x + 16^y + 16^z \ge 2^{x+y+z}(2^x + 2^y + 2^z) \tag{4}$$

By iphotessis:

$$16^x + 16^y + 16^z \le 2^{x+y+z}(2^x + 2^y + 2^z) \tag{5}$$

By $(4); (5)$:

$$16^x + 16^y + 16^z = 2^{x+y+z}(2^x + 2^y + 2^z)$$

Equality holds for: $2^x = 2^y = 2^z = 1 \Rightarrow x = y = z = 0$. □

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