PP44907

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}(s-a)(a+b)(a+c) = s(s^2+5r^2+8Rr)$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}(s-a)(a+b)(a+c) = \sum_{cyc}(s-a)(2s-b)(2s-c) =$$

$$= \sum_{cyc}(s-a)(4s^2-2s(b+c)+bc) =$$

$$= \sum_{cyc}(s-a)(4s^2-2s(2s-a)+bc) =$$

$$= \sum_{cyc}(s-a)(4s^2-4s^2+2sa+bc) =$$

$$= \sum_{cyc}(s-a)(2sa+bc) =$$

$$= s\sum_{cyc}(2sa+bc) - \sum_{cyc}(2sa^2+abc) =$$

$$= 2s^2\sum_{cyc}a + s\sum_{cyc}bc - 2s\sum_{cyc}a^2 - 3abc =$$

$$= 2s^2\cdot 2s + s(s^2+r^2+4Rr) - 4s(s^2-r^2-4Rr) - 3abc =$$
$$= 4s^3+s^3+sr^2+4Rrs-4s^3+4sr^2+16Rrs-12Rrs =$$
$$= s^3+5sr^2+8Rrs =$$
$$= s(s^2+5r^2+8Rr)$$

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