PP45090

MIHÁLY BENCZE - ROMANIA

Find a closed form:

$$\Omega = \int_{-1}^{1} \frac{(x^{4n} + x^{2n} + 1)dx}{e^x + 1}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\text{Let } y = -x \Rightarrow dx = -dy$$

$$x = 1 \Rightarrow y = -1$$

$$x = -1 \Rightarrow y = 1$$

$$\Omega = \int_{+1}^{-1} \frac{(-y)^{4n} + (-y)^{2n} + 1}{e^{-y} + 1} \cdot (-dy) =$$

$$= \int_{-1}^{1} \frac{y^{4n} + y^{2n} + 1}{\frac{1+e^y}{e^y}} dy = \int_{-1}^{1} \frac{e^x(x^{4n} + x^{2n} + 1)}{1 + e^x} dx$$

$$\Omega + \Omega = \int_{-1}^{1} \frac{x^{4n} + x^{2n+1} + 1}{e^x + 1} dx + \int_{-1}^{1} \frac{e^x(x^{4n} + x^{2n} + 1)}{e^x + 1} dx$$

$$2\Omega = \int_{-1}^{1} \frac{(e^x + 1)(x^{4n} + x^{2n} + 1)}{e^x + 1} dx$$

$$\Omega = \frac{1}{2}\left(\int_{-1}^{1} x^{4n} dx + \int_{-1}^{1} x^{2n} dx + \int_{-1}^{1} dx\right)$$

$$\Omega = \frac{1}{2}\Big(\frac{1^{4n+1}}{4n+1} - \frac{(-1)^{4n+1}}{4n+1}\Big) + \frac{1}{2}\Big(\frac{1^{2n+1}}{2n+1} - \frac{(-1)^{2n+1}}{2n+1}\Big) + \frac{1}{2}(1 - (-1))$$

$$\Omega = \frac{1}{4n+1} + \frac{1}{2n+1} + 1 = \frac{2n + 1 + 4n + 1 + (2n+1)(4n+1)}{(2n+1)(4n+1)}$$

$$\Omega = \frac{6n + 2 + 8n^2 + 2n + 4n + 1}{(2n+1)(4n+1)} = \frac{8n^2 + 12n + 3}{(2n+1)(4n+1)}$$

□

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