PP45104

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{r_a^2 + (4R+r)r_a}{r_b + r_c} > 3s$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{r_a^2 + (4R+r)r_a}{r_b + r_c} = \sum_{cyc} \frac{r_a^2}{r_b + r_c} + (4R+r)\sum_{cyc} \frac{r_a}{r_b + r_c} \ge$$

$$\overset{\text{BERGSTRÖM}}{\ge} \frac{(r_a + r_b + r_c)^2}{2(r_a + r_b + r_c)} + (4R+r)\sum_{cyc} \frac{r_a^2}{r_a r_b + r_c r_a} \ge$$

$$\overset{\text{BERGSTRÖM}}{\ge} \frac{r_a + r_b + r_c}{2} + (4R+r) \cdot \frac{(r_a + r_b + r_c)^2}{2(r_a r_b + r_b r_c + r_c r_a)} =$$

$$= \frac{4R+r}{2} + \frac{(4R+r)^3}{2s^2} \overset{\text{AM-GM}}{>}$$

$$> 2\sqrt{\frac{(4R+r)^4}{4s^2}} = 2 \cdot \frac{(4R+r)^2}{2s} \ge$$

$$\overset{\text{DOUCET}}{\ge} \frac{(s\sqrt{3})^2}{s} = 3s$$

□

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