PP45139

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$R\sum_{cyc}\frac{h_a}{r_a}=2r\sum_{cyc}\frac{r_ar_b}{h_ah_b}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$R\sum_{cyc}\frac{h_a}{r_a}=2r\sum_{cyc}\frac{r_ar_b}{h_ah_b}$$

$$R\sum_{cyc}\frac{\frac{2F}{a}}{\frac{F}{s-a}}=2r\sum_{cyc}\frac{\frac{F}{s-a}\cdot\frac{F}{s-b}}{\frac{2F}{a}\cdot\frac{2F}{b}}$$

$$2R\sum_{cyc}\frac{s-a}{a}=\frac{2r}{4}\sum_{cyc}\frac{ab}{(s-a)(s-b)}$$

$$\frac{2R}{abc}\sum_{cyc}bc(s-a)=\frac{r}{2(s-a)(s-b)(s-c)}\sum_{cyc}ab(s-c)$$

$$\frac{2R}{4RF}\left(s\sum_{cyc}bc-3abc\right)=\frac{rs}{2F^2}\left(s\sum_{cyc}ab-3abc\right)$$

$$\frac{2}{4F}=\frac{F}{2F^2}$$

$$\frac{1}{2F}=\frac{1}{2F}$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com