PP45152

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{(a+b)w_c}{\cos\frac{A}{2}\cos\frac{B}{2}} = 8Rs$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc} \frac{(a+b)w_c}{\cos\frac{A}{2}\cos\frac{B}{2}} = \sum_{cyc} \frac{(a+b)\cdot\frac{2}{a+b}\sqrt{abs(s-c)}}{\sqrt{\frac{s(s-a)}{bc}}\cdot\sqrt{\frac{s(s-b)}{ac}}} =$$

$$= 2\sum_{cyc}\sqrt{\frac{abs(s-c)\cdot bc\cdot ac}{s^2(s-a)(s-b)}} =$$

$$= 2abc\sum_{cyc}\sqrt{\frac{s-c}{s(s-a)(s-b)}} =$$

$$= 2\cdot 4RF\sum_{cyc}\sqrt{\frac{(s-c)^2}{s(s-a)(s-b)(s-c)}} =$$

$$= 8RF\sum_{cyc}\frac{s-c}{\sqrt{s(s-a)(s-b)(s-c)}} =$$

$$= \frac{8RF}{F}\sum_{cyc}(s-c) = 8R\left(\sum_{cyc}s - \sum_{cyc}c\right) =$$

$$= 8R(3s-2s) = 8Rs$$

□

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