PP45160

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{w_a w_b \cos \frac{C}{2}}{a+b} = \frac{4s^2 r}{s^2 + r^2 + 2Rr}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc} \frac{w_a w_b \cos \frac{C}{2}}{a+b} =$$

$$= \sum_{cyc} \frac{\frac{2}{b+c}\sqrt{bcs(s-a)} \cdot \frac{2}{a+c}\sqrt{acs(s-b)} \cdot \sqrt{\frac{s(s-c)}{ab}}}{a+b} =$$

$$= \frac{4}{(a+b)(b+c)(c+a)} \sum_{cyc} sc\sqrt{s(s-a)(s-b)(s-c)} =$$

$$= \frac{4sF}{2s(s^2 + r^2 + 2Rr)} \sum_{cyc} c =$$

$$= \frac{2F}{s^2 + r^2 + 2Rr} \cdot 2s = \frac{4sF}{s^2 + r^2 + 2Rr} =$$

$$= \frac{4s \cdot sr}{s^2 + r^2 + 2Rr} = \frac{4s^2 r}{s^2 + r^2 + 2Rr}$$

$\square$

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