PP45179

MIHÁLY BENCZE - ROMANIA

In all acute triangles ABC holds:

$$\frac{1}{8R}\sum_{cyc}(a-b)^2 \le R-2r$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\frac{1}{8R}\sum_{cyc}(a-b)^2 = \frac{1}{8R}\sum_{cyc}(a^2-2ab+b^2) =$$

$$= \frac{1}{8R}\left(2\sum_{cyc}a^2 - 2\sum_{cyc}ab\right) =$$

$$= \frac{1}{4R}(2s^2-2r^2-8Rr-s^2-r^2-4Rr) =$$

$$= \frac{1}{4R}(s^2-12Rr-3r^2) \overset{\text{GERRETSEN}}{\le}$$

$$\le \frac{1}{4R}(4R^2+4Rr+3r^2-12Rr-3r^2) =$$

$$= \frac{1}{4R}(4R^2-8Rr) = R-2r$$

Equality holds for $a=b=c$.

□

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