PP45193

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$1.\ a^2r_br_c = 4s^2r(R+r)$$

$$2.\ a^2h_bh_c = \frac{2s^2r}{R}(s^2-6Rr-3r^2)$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$1.\ \sum_{cyc} a^2r_br_c = \sum_{cyc} a^2\cdot\frac{F}{s-b}\cdot\frac{F}{s-c} =$$

$$= F^2\sum_{cyc}\frac{a^2}{(s-b)(s-c)} = \frac{F^2}{(s-a)(s-b)(s-c)}\cdot\sum_{cyc} a^2(s-a) =$$

$$= \frac{F^2 s}{s(s-a)(s-b)(s-c)}\cdot\left(s\sum_{cyc}a^2-\sum_{cyc}a^3\right) =$$

$$= \frac{F^2 s}{F^2}(s\cdot 2(s^2-r^2-4Rr)-2s(s^2-3r^2-6Rr)) =$$

$$= s\cdot 2s(s^2-r^2-4Rr-s^2+3r^2+6Rr) =$$

$$= 2s^2(2r^2+2Rr) = 4s^2r(R+r)$$

$$2.\ \sum_{cyc} a^2h_bh_c = \sum_{cyc} a^2\cdot\frac{2F}{b}\cdot\frac{2F}{c} = 4F^2\sum_{cyc}\frac{a^2}{bc} =$$

$$= \frac{4F^2}{abc}\sum_{cyc}a^3 = \frac{4F^2}{4RF}\cdot 2s(s^2-3r^2-6Rr) =$$

$$= \frac{2s\cdot rs}{R}(s^2-3r^2-6Rr) =$$

$$= \frac{2s^2r}{R}(s^2-6Rr-3r^2)$$

□

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