PP45197

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\sin\frac{A}{2}\cos\frac{B-C}{2}=1+\frac{r}{R}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\sin\frac{A}{2}\cos\frac{B-C}{2}=\sum_{cyc}\frac{1}{2}\Big(\sin\Big(\frac{A}{2}+\frac{B-C}{2}\Big)+\sin\Big(\frac{A}{2}-\frac{B-C}{2}\Big)\Big)=$$

$$=\frac{1}{2}\sum_{cyc}\sin\frac{A+B-C}{2}+\frac{1}{2}\sum_{cyc}\sin\frac{A+C-B}{2}=$$

$$=\frac{1}{2}\sum_{cyc}\sin\frac{\pi-2C}{2}+\frac{1}{2}\sum_{cyc}\sin\frac{\pi-2B}{2}=$$

$$=\frac{1}{2}\sum_{cyc}\cos C+\frac{1}{2}\sum_{cyc}\cos B=\frac{1}{2}\cdot 2\sum_{cyc}\cos A=$$

$$=\sum_{cyc}\Big(2\cos^2\frac{A}{2}-1\Big)=2\sum_{cyc}\cos^2\frac{A}{2}-3=$$

$$=2\sum_{cyc}\frac{s(s-a)}{bc}-3=\frac{2s}{abc}\sum_{cyc}a(s-a)-3=$$

$$=\frac{2s}{4Rrs}\left(s\sum_{cyc}a-\sum_{cyc}a^2\right)-3=$$

$$=\frac{1}{2Rr}(s\cdot 2s-2s^2+2r^2+8Rr)-3=$$

$$=\frac{1}{2Rr}\cdot 2r(r+4R)-3=\frac{1}{R}(r+4R)-3=$$

$$=\frac{r+4R-3R}{R}=\frac{R+r}{R}=1+\frac{r}{R}$$

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