PP45216

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If $x \in \mathbb{R}$ then:

$$\frac{1}{3} \leq \frac{\sin^4 x}{1+\sin^2 x} + \frac{\cos^4 x}{1+\cos^2 x} \leq \frac{1}{2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\frac{\sin^4 x}{1+\sin^2 x} + \frac{\cos^4 x}{1+\cos^2 x} = \frac{(\sin^2 x)^2}{1+\sin^2 x} + \frac{(\cos^2 x)^2}{1+\cos^2 x} \geq$$

$$\overset{\text{BERGSTRÖM}}{\geq} \frac{(\sin^2 x + \cos^2 x)^2}{1+\sin^2 x + 1 + \cos^2 x} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$\frac{\sin^4 x}{1+\sin^2 x} + \frac{\cos^4 x}{1+\cos^2 x} \leq \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{\sin^4 x + \sin^4 \cos^2 x + \cos^4 x + \cos^4 x \sin^2 x}{1+\cos^2 x + \sin^2 x + \sin^2 x \cos^2 x} \leq \frac{1}{2}$$

$$\frac{\sin^4 x + \cos^4 x + \sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)}{2+\sin^2 x \cos^2 x} \leq \frac{1}{2}$$

$$\frac{(\sin^2 x + \cos^2 x) - 2\sin^2 x \cos^2 x + \sin^2 x \cos^2 x}{2+\sin^2 x \cos^2 x} \leq \frac{1}{2}$$

$$2 - 2\sin^2 x \cos^2 x \leq 2 + \sin^2 x \cos^2 x$$

$$2\sin^2 x \cos^2 x \geq 0 \text{ (True)}$$

□

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