PP45225

MIHÁLY BENCZE - ROMANIA

Solve for real numbers:

$$\begin{cases} 3x_1^4+1=4x_2^3 \\ 3x_2^4+1=4x_3^3 \\ -------- \\ 3x_n^4+1=4x_1^3 \end{cases}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$4x_2^3=3x_1^4+1=x_1^4+x_1^4+x_1^4+1\overset{\text{AM-GM}}{\geq}$$

$$\geq 4\sqrt[4]{x_1^4\cdot x_1^4\cdot x_1^4\cdot 1}=4x_1^3$$

$$4x_2^3\geq 4x_1^3\Rightarrow x_2^3\geq x_1^3\Rightarrow x_2\geq x_1$$

Analogous:

$$x_3\geq x_2; x_4\geq x_3;\ldots;x_n\geq x_{n-1};x_1\geq x_n$$

$$x_n\leq x_1\leq x_2\leq x_3\leq\ldots\leq x_n\leq x_1$$

$$x_1=x_2=x_3=\ldots=x_n=\alpha$$

$$3\alpha^4+1=4\alpha^3\Rightarrow 3\alpha^4-4\alpha^3+1=0$$

$$3\alpha^4-3\alpha^3-(\alpha^3-1)=0$$

$$3\alpha^3(\alpha-1)-(\alpha-1)(\alpha^2+\alpha+1)=0$$

$$(\alpha-1)(3\alpha^3-\alpha^2-\alpha-1)=0$$

$$(\alpha-1)(3\alpha^3-3\alpha^2+2\alpha^2-2\alpha+\alpha-1)=0$$

$$(\alpha-1)[3\alpha^2(\alpha-1)+2\alpha(\alpha-1)+(\alpha-1)]=0$$

$$(\alpha-1)^2(3\alpha^2+2\alpha+1)=0$$

$$(\alpha-1)^2=0\Rightarrow\alpha-1=0\Rightarrow\alpha=1$$

$$x_1=x_2=\ldots=x_n=1$$

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