PP45256

MIHÁLY BENCZE - ROMANIA

If $a, b, c > 0$ then:

$$\left(\frac{ab}{c}\right)^c \cdot \left(\frac{bc}{a}\right)^a \cdot \left(\frac{ca}{b}\right)^b \le \left(\frac{ab+bc+ca}{a+b+c}\right)^{a+b+c}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\left(\frac{ab}{c}\right)^c \cdot \left(\frac{bc}{a}\right)^a \cdot \left(\frac{ca}{b}\right)^b = \frac{a^c \cdot b^c \cdot b^a \cdot c^a \cdot c^b \cdot a^b}{c^c \cdot a^a \cdot b^b}$$

$$= b^{b+c-a} \cdot b^{c+a-b} \cdot c^{a+b-c} \overset{\text{WEIGHTED AM-GM}}{\le}$$

$$\le \left(\frac{a(b+c-a)+b(c+a-b)+c(a+b-c)}{(b+c-a)+(c+a-b)+(a+b-c)}\right)^{(b+c-a)+(c+a-b)+(a+b-c)} =$$

$$= \left(\frac{2(ab+bc+ca)-(a^2+b^2+c^2)}{a+b+c}\right)^{a+b+c} \le$$

$$\le \left(\frac{2(ab+bc+ca)-(ab+bc+ca)}{a+b+c}\right)^{a+b+c} =$$

$$= \left(\frac{ab+bc+ca}{a+b+c}\right)^{a+b+c}$$

Equality holds for $a = b = c$.

□

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