PP45288

MIHÁLY BENCZE - ROMANIA

If $x \in [0,1]$ then:

$$1 + \arcsin x \geq \sqrt{\frac{1-x}{1+x}}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\text{Let be } f : [0,1] \to \mathbb{R}; f(x) = 1 + \arcsin x - \sqrt{\frac{1-x}{1+x}}$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{\left(\frac{1-x}{1+x}\right)'}{2\sqrt{\frac{1-x}{1+x}}}$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{(1-x)'(1+x) - (1-x)(1+x)'}{2(1+x)^2\sqrt{\frac{1-x}{1+x}}}$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{-1-x-1+x}{2(1+x)^2} \cdot \sqrt{\frac{1+x}{1-x}} > 0$$

$$\Rightarrow f \text{ increasing } \Rightarrow \min_{x \in [0,1]} f(x) = f(0) = 0$$

$$f(x) \geq 0; (\forall) x \in [0,1]$$

$$1 + \arcsin x - \sqrt{\frac{1-x}{1+x}} \geq 0$$

$$1 + \arcsin x \geq \sqrt{\frac{1-x}{1+x}}$$

□

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