PP45306

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\cos\frac{A}{2}\geq\frac{(R+2r)s}{2R^2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc}\cos\frac{A}{2}\overset{\text{AM-GM}}{\geq}3\cdot\sqrt[3]{\prod_{cyc}\cos\frac{A}{2}}=3\cdot\sqrt[3]{\frac{s}{4R}}$$

Remains to prove that:

$$3\sqrt[3]{\frac{s}{4R}}\geq\frac{(R+2r)s}{2R^2}\Leftrightarrow 27\cdot\frac{s}{4R}\geq\frac{(R+2r)^3s^3}{8R^6}$$

$$\Leftrightarrow 27\geq\frac{(R+2r)^3\cdot s^2}{2R^5}\Leftrightarrow s^2(R+2r)^3\leq 54R^5 \text{ (to prove)}$$

$$s^2(R+2r)^3\overset{\text{MITRINOVIC}}{\leq}\left(\frac{3\sqrt{3}}{2}R\right)^2\cdot(R+2r)^3\leq$$

$$\overset{\text{EULER}}{\leq}\frac{27}{4}R^2\cdot(R+r)^3=\frac{27R^2}{4}\cdot 8R^3=54R^5$$

Equality holds for $a=b=c$. □

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