PP45323

MIHÁLY BENCZE - ROMANIA

In all acute triangles ABC holds:

$$\prod_{cyc}\Big(\sqrt{\frac{a}{\cos A}}+\sqrt{\frac{b}{\cos B}}\Big)\geq 16\sqrt{2abc}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\prod_{cyc}\Big(\sqrt{\frac{a}{\cos A}}+\sqrt{\frac{b}{\cos B}}\Big)\overset{\text{AM-GM}}{\geq}\prod_{cyc}2\cdot\sqrt[4]{\frac{ab}{\cos A\cos B}}=$$

$$=8\sqrt[4]{(\frac{abc}{\cos A\cos B\cos C})^2}=8\sqrt{\frac{abc}{\cos A\cos B\cos C}}\geq$$

$$\geq 16\sqrt{2abc}\Leftrightarrow$$

$$\Leftrightarrow\frac{1}{\sqrt{\cos A\cos B\cos C}}\geq 2\sqrt{2}\Leftrightarrow$$

$$\Leftrightarrow\cos A\cos B\cos C\leq\frac{1}{8}$$

$$\frac{a^2+b^2+c^2-8R^2}{8R^2}\leq\frac{1}{8}$$

$$a^2+b^2+c^2-8R^2\leq R^2$$

$$a^2+b^2+c^2\leq 9R^2$$

(Leibniz)

Equality holds for $a=b=c$. □

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