PP45457

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\frac{r_a+r_b}{c}+\frac{r_b+r_c}{a}+\frac{r_c+r_a}{b}=\frac{(r_a+r_b)(r_b+r_c)(r_a+r_a)}{abc}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\frac{r_a+r_b}{c}=\prod_{cyc}\frac{r_a+r_b}{c}$$

$$\sum_{cyc}\frac{\frac{F}{s-a}+\frac{F}{s-b}}{c}=\prod_{cyc}\frac{\frac{F}{s-a}+\frac{F}{s-b}}{c}$$

$$F\sum_{cyc}\frac{s-b+s-a}{c(s-a)(s-b)}=F^3\prod_{cyc}\frac{s-b+s-a}{c(s-a)(s-b))}$$

$$F\sum_{cyc}\frac{a+b+c-a-b}{c(s-a)(s-b)}=F^3\prod_{cyc}\frac{a+b+c-a-b}{c(s-a)(s-b)}$$

$$\sum_{cyc}\frac{1}{(s-a)(s-b)}=F^2\prod_{cyc}\frac{1}{(s-a)(s-b)}$$

$$\frac{1}{(s-a)(s-b)(s-c)}\sum_{cyc}(s-c)=\frac{s(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}$$

$$\sum_{cyc}s-\sum_{cyc}c=s$$

$$3s-2s=s$$

$$s=s$$

□

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