PP45458

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\sqrt{r_a+r_b}\le s\sqrt{\frac{2}{r}}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\sqrt{r_a+r_b}=\sum_{cyc}(1\cdot\sqrt{r_a+r_b})\overset{\text{CBS}}{\le}$$

$$\le\sqrt{(1^2+1^2+1^2)(r_a+r_b+r_b+r_c+r_c+r_a)}=$$

$$=\sqrt{6(r_a+r_b+r_c)}=\sqrt{6(4R+r)}\le s\sqrt{\frac{2}{r}}\Leftrightarrow$$

$$\Leftrightarrow 6(4R+r)\le s\left(\sqrt{\frac{2}{r}}\right)^2\Leftrightarrow$$

$$\Leftrightarrow 6(4R+r)\le s^2\cdot\frac{2}{r}\Leftrightarrow 12Rr+3r^2\le s^2\text{ (to prove)}$$

$$s^2\overset{\text{GERRETSEN}}{\ge}16Rr-5r^2\ge 12Rr+3r^2$$

$$4Rr\ge 8r^2$$

$$R\ge 2r\text{ (Euler)}$$

Equality holds for $a=b=c$.

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