PP45461

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:
$$\sum_{cyc} bc(r_a + r_b)(r_a + r_c) \geq (s^2 + r^2 + 4Rr)^2$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} bc(r_a + r_b)(r_a + r_c) =$$

$$= \sum_{cyc} bc\Big(\frac{F}{s-a} + \frac{F}{s-b}\Big)\Big(\frac{F}{s-a} + \frac{F}{s-c}\Big) =$$

$$= F^2 \sum_{cyc} bc \cdot \frac{s-b+s-a}{(s-a)(s-b)} \cdot \frac{s-c+s-a}{(s-a)(s-c)} =$$

$$= \frac{F^2}{(s-a)(s-b)(s-c)} \sum_{cyc} \frac{bc \cdot c \cdot b}{s-a} =$$

$$= \frac{F^2 s}{s(s-a)(s-b)(s-c)} \sum_{cyc} \frac{(bc)^2}{s-a} \overset{\text{BERGSTRÖM}}{\geq}$$

$$\geq \frac{F^2 s}{F^2} \cdot \frac{(bc+ca+ab)^2}{s-a+s-b+s-c} =$$

$$= s \cdot \frac{(s^2+r^2+4Rr)^2}{s} = (s^2+r^2+4Rr)^2$$

Equality holds for $a = b = c$.

$\square$

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