PP45467

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\left(\frac{m_a}{a}\right)^2 \geq \frac{9r}{2R}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\left(\frac{m_a}{a}\right)^2 \overset{\text{AM-GM}}{\geq} 3\cdot\sqrt[3]{\frac{(m_am_bm_c)^2}{(abc)^2}} \geq$$

$$\geq 3\cdot\sqrt[3]{\frac{(r_ar_br_c)^2}{(4RF)^2}} = 3\cdot\sqrt[3]{\frac{F^6}{16R^2F^2(s-a)^2(s-b)^2(s-c)^2}} =$$

$$= \frac{3}{2}\sqrt[3]{\frac{F^4}{2R^2(s-a)^2(s-b)^2(s-c)^2}} =$$

$$= \frac{3}{2}\sqrt[3]{\frac{s^2(s-a)^2(s-b)^2(s-c)^2}{2R^2(s-a)^2(s-b)^2(s-c)^2}} = \frac{3}{2}\sqrt[3]{\frac{s^2}{2R^2}} \geq$$

$$\overset{\text{MITRINOVIC}}{\geq} \frac{3}{2}\sqrt[3]{\frac{27r^2}{2R^2}} = \frac{3}{2}\cdot 3\sqrt[3]{\frac{r^2}{2R^2}} =$$

$$= \frac{9}{2}\sqrt[3]{\frac{r^3}{2R^2r}} = \frac{9r}{2}\cdot\sqrt[3]{\frac{1}{2R^2r}} \overset{\text{EULER}}{\geq}$$

$$\geq \frac{9r}{2}\cdot\sqrt[3]{\frac{1}{2R^2\cdot\frac{R}{2}}} = \frac{9r}{2}\cdot\frac{1}{R} = \frac{9r}{2R}$$

Equality holds for $a = b = c$. □

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