PP45471

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$s^2 + r^2 - 8Rr \geq 4r(2R - r) \geq 6Rr$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$s^2 + r^2 - 8Rr \overset{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 + r^2 - 8Rr =$$

$$= 8Rr - 4r^2 = 4r(2R - r)$$

$$4r(2R - r) \geq 6Rr \Leftrightarrow 8Rr - 4r^2 \geq 6Rr \Leftrightarrow$$

$$\Leftrightarrow 2R \geq 4r^2 \Leftrightarrow R \geq 2r \text{ (EULER)}$$

Equality holds for $a = b = c$. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com