

PP45480

MIHÁLY BENCZE ROMANIA

If $a, b, c > 0$ then:

$$\sum_{cyc} \frac{1}{a^{15} + b^3 c^3 (b^9 + c^9)} \leq \frac{1}{a^5 b^5 c^5}$$

Solution by Daniel Sitaru, Claudia Nănuți.

$$\begin{aligned} \sum_{cyc} \frac{1}{a^{15} + b^3 c^3 (b^9 + c^9)} &= \sum_{cyc} \frac{1}{a^{15} + b^{12} c^3 + b^3 c^{12}} \leq \\ &\stackrel{\text{AM-GM}}{\leq} \sum_{cyc} \frac{1}{3 \sqrt[3]{a^{15} \cdot b^{12} c^3 \cdot b^3 c^{12}}} = \\ &= \sum_{cyc} \frac{1}{3 \sqrt[3]{(abc)^{15}}} = \frac{1}{3} \sum_{cyc} \frac{1}{(abc)^5} = \\ &= \frac{1}{3} \cdot \frac{3}{(abc)^5} = \frac{1}{a^5 b^5 c^5} \end{aligned}$$

Equality holds for $a = b = c$. □

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