**PP45523**

MIHÁLY BENCZE - ROMANIA

In all triangles $ABC$ holds:

$$1.\ \sum_{cyc} m_a^2 = \frac{3}{4}a^2$$

$$2.\ \sum_{cyc} m_a^4 = \frac{9}{16}\sum_{cyc} a^4$$

$$3.\ \sum_{cyc} m_a^2 m_b^2 = \frac{9}{16}\sum_{cyc} a^2b^2$$

*Solution by Daniel Sitaru, Claudia Nănuţi.*

$$1.\ \sum_{cyc} m_a^2 = \sum_{cyc}\left(\frac{1}{2}(b^2+c^2) - \frac{1}{4}a^2\right) =$$

$$= \frac{1}{2}\sum_{cyc} a^2 + \frac{1}{2}\sum_{cyc} a^2 - \frac{1}{4}\sum_{cyc} a^2 =$$

$$=\left(\frac{1}{2}+\frac{1}{2}-\frac{1}{4}\right)\sum_{cyc} a^2 = \frac{3}{4}\sum_{cyc} a^2$$

$$2.\ \sum_{cyc} m_a^4 = \sum_{cyc}\left(\frac{2(b^2+c^2)-a^2}{4}\right)^2 =$$

$$= \frac{1}{16}\sum_{cyc}(4(b^2+c^2) + a^4 - 4(b^2+c^2)a^2) =$$

$$= \frac{1}{4}\sum_{cyc}(b^4+c^4+2b^2c^2) + \frac{1}{16}\sum_{cyc} a^4 - \frac{1}{4}\sum_{cyc}(a^2b^2+a^2c^2) =$$

$$= \frac{1}{2}\sum_{cyc} a^4 + \frac{1}{2}\sum_{cyc} a^2b^2 + \frac{1}{16}\sum_{cyc} a^4 - \frac{1}{2}\sum_{cyc} a^2b^2 =$$

$$=\left(\frac{1}{2}+\frac{1}{16}\right)\sum_{cyc} a^4 = \frac{9}{16}\sum_{cyc} a^4$$

$$3.\ \left(\sum_{cyc} m_a^2\right)^2 = \sum_{cyc} m_a^4 + 2\sum_{cyc} m_a^2 m_b^2$$

$$\left(\frac{3}{4}\sum_{cyc} a^2\right)^2 = \frac{9}{16}\sum_{cyc} a^4 + 2\sum_{cyc} m_a^2 m_b^2$$

$$\frac{9}{16}\sum_{cyc} a^4 + \frac{9}{8}\sum_{cyc} a^2b^2 = \frac{9}{16}\sum_{cyc} a^4 + 2\sum_{cyc} a^2b^2$$

$$2\sum_{cyc} m_a^2 m_b^2 = \frac{9}{8}\sum_{cyc} a^2 b^2$$
$$\sum_{cyc} m_a^2 m_b^2 = \frac{9}{16}\sum_{cyc} a^2 b^2$$

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