PP45639

MIHÁLY BENCZE - ROMANIA

If $a, b > 0$ and $ab = 1$ then:

$$a^2 + b^2 \geq a + b$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$ab = 1 \Rightarrow b = \frac{1}{a}$$

We must prove that:

$$a^2 + \frac{1}{a^2} \geq a + \frac{1}{a}$$
$$\frac{a^4 + 1}{a^2} \geq \frac{a^2 + 1}{a}$$
$$a^4 + 1 \geq a^3 + a$$
$$a^4 - a^3 - (a - 1) \geq 0$$
$$a^3(a - 1) - (a - 1) \geq 0$$
$$(a - 1)(a^3 - 1) \geq 0$$
$$(a - 1)^2(a^2 + a + 1) \geq 0 \text{ (True)}$$

Equality holds for $a = b = 1$.

□

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