PP45653

MIHÁLY BENCZE- ROMANIA

If $a, b > 0$ and $a + b = 2$ then:

$$a^2 + b^2 + 2(a-1)^2(b-1)^2 \geq 2$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\text{Denote: } x = a - 1; y = b - 1 \Rightarrow$$

$$x + y = a - 1 + b - 1 = a + b - 2 = 0 \Rightarrow y = -x$$

We must prove that:

$$(x+1)^2 + (y+1)^2 + 2x^2y^2 \geq 2$$

$$(x+1)^2 + (-x+1)^2 + 2x^2 \cdot (-x)^2 - 2 \geq 0$$

$$x^2 + 2x + 1 + x^2 - 2x + 1 + 2x^4 - 2 \geq 0$$

$$2x^4 + 2x^2 \geq 0$$

$$2x^2(x^2+1) \geq 0 \text{ (True)}$$

Equality holds for $x = y = 0 \Rightarrow a = b = 1$.

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com