PP45655

MIHÁLY BENCZE - ROMANIA

If $a, b > 0$ and $a + b = 1$ then:

$$(2a^2 + b^2)(a^2 + 2b^2) \leq 2$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$a + b = 1 \Rightarrow a = 1 - b \geq 0 \Rightarrow b \leq 1$$

$$b = 1 - a \geq 0 \Rightarrow a \leq 1 \Rightarrow a - 1 \leq 0$$

$$(2a^2 + b^2)(a^2 + 2b^2) \leq 2$$

$$(2a^2 + (1 - a)^2)(a^2 + 2(1 - a)^2) \leq 2$$

$$(3a^2 - 2a + 1)(3a^2 - 4a + 2) \leq 2$$

$$\begin{aligned} 9a^4 - 12a^3 + 6a^2 - \\ -6a^3 + 8a^2 - 4a + \\ +3a^2 - 4a + 2 \leq 2 \end{aligned}$$

$$9a^4 - 18a^3 + 17a^2 - 8a \leq 0$$

$$9a^3 - 18a^2 + 17a - 7 \leq 0$$

$$9a^3 - 9a^2 - 9a^2 + 9a + 8a - 8 \leq 0$$

$$9a^2(a - 1) - 9a(a - 1) + 8(a - 1) \leq 0$$

$$(a - 1)(9a^2 - 9a + 8) \leq 0$$

$$(a - 1)\left[9\left(a - \frac{1}{2}\right)^2 + \frac{23}{4}\right] \leq 0$$

$$a - 1 \leq 0$$

$$a \leq 1 \text{ (True)}$$

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