PP45682

MIHÁLY BENCZE - ROMANIA

If $a, b, c, d > 0$ then:

$$\sum_{cyc} \frac{a+b+c}{(\sqrt{a+b+c}+\sqrt{d})^2} \geq \frac{3}{2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.
Lemma: If $a, b \geq 0$ then:

$$(\sqrt{a}+\sqrt{b})^2 \leq 2(a+b)$$

Proof.

$$(\sqrt{a}+\sqrt{b})^2 \leq 2(a+b) \Leftrightarrow a+b+2\sqrt{ab} \leq 2(a+b)$$

$$\Leftrightarrow a-2\sqrt{ab}+b \geq 0 \Leftrightarrow (\sqrt{a}-\sqrt{b})^2 \geq 0$$

$$\sum_{cyc} \frac{a+b+c}{(\sqrt{a+b+c}+\sqrt{d})^2} \overset{\text{Lemma}}{\geq} \sum_{cyc} \frac{a+b+c}{2(a+b+c+d)} =$$

$$= \frac{1}{2(a+b+c+d)} \cdot \sum_{cyc}(a+b+c) =$$

$$= \frac{1}{2(a+b+c)} \cdot 3(a+b+c+d) = \frac{3}{2}$$

□

Equality holds for $a = b = c = d$. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania
Email address: dansitaru63@yahoo.com