PP45711

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\frac{R}{r} \geq \frac{s^2 - r^2 - 4Rr}{r(4R+r)} \geq 2$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\frac{s^2 - r^2 - 4Rr}{r(4R+r)} \geq 2 \Leftrightarrow s^2 - r^2 - 4Rr \geq 8Rr + 2r^2$$

$$\Leftrightarrow s^2 \geq 12Rr + 3r^2 \text{ (to prove)}$$

$$s^2 \overset{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 \geq 12Rr + 3r^2 \Leftrightarrow$$

$$\Leftrightarrow 4Rr \geq 8r^2 \Leftrightarrow R \geq 2r \text{ (Euler)}$$

$$\Leftrightarrow 3r^2 - r^2 \leq 5Rr - 4Rr \Leftrightarrow Rr \geq 2r^2$$

$$\Leftrightarrow R \geq 2r \text{ (Euler)}$$

Equality holds for $a = b = c$.

□

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