

**PP45783**

MIHÁLY BENCZE - ROMANIA

If  $a, b > 0$  then:

$$9(a^2 + b^2)(a + b)^2 \geq 8ab(2a + b)(a + 2b)$$

*Solution by Daniel Sitaru and Claudia Nănuți.*

$$9(a^2 + b^2)(a + b)^2 \stackrel{\text{AM-GM}}{\geq} 9 \cdot 2ab(a + b)^2$$

Remains to prove:

$$\begin{aligned} 18ab(a + b)^2 &\geq 8ab(2a + b)(a + 2b) \\ 9(a^2 + b^2 + 2ab) &\geq 4(2a^2 + 2b^2 + 5ab) \\ 9a^2 - 8a^2 + 9b^2 - 8b^2 + 18ab - 20ab &\geq 0 \\ a^2 - 2ab + b^2 &\geq 0 \\ (a - b)^2 &\geq 0 \end{aligned}$$

Equality holds for  $a = b$ . □

MATHEMATICS DEPARTMENT, NATIONAL ECONOMIC COLLEGE "THEODOR COSTESCU", DROBETA  
TURNU - SEVERIN, ROMANIA  
*Email address:* dansitaru63@yahoo.com