PP45797

MIHÁLY BENCZE - ROMANIA

If $x \in (0, \pi)$ then:

$$2\sqrt[4]{\sin x} + \frac{1}{2\sin x} \geq \frac{5}{2}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\text{Denote } \sin x = a \in (0,1)$$

$$\text{Let be } f : (0,1) \to \mathbb{R}; f(a) = 2\sqrt[4]{a} + \frac{1}{2a^2}$$

$$f'(a) = \frac{1}{2\sqrt[4]{a^3}} - \frac{1}{2a^2}$$

$$f'(a) = 0 \Rightarrow a^2 = \sqrt[4]{a^3} \Rightarrow a^8 - a^3 = 0$$

$$\Rightarrow a^3(a^5 - 1) = 0 \Rightarrow a = 1$$

$$f \text{ decreasing; } \min_{a \in (0,1)} f(a) = \lim_{\substack{a \to 1 \\ a < 1}} f(a) = 2 + \frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow f(a) \geq \frac{5}{2}; (\forall) a \in (0,1)$$

Equality holds for: $a = 1 \Rightarrow x = \frac{5}{2}$. □

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