PP46675

MIHÁLY BENCZE - ROMANIA

If $a, b, c > 0$ then:

$$\left(\sum_{cyc} a\right)\left(\sum_{cyc} \frac{1}{a}\right) = 1 + \frac{(a+b)(b+c)(c+a)}{abc} \geq 9$$

Solution by Daniel Sitaru.

$$\left(\sum_{cyc} a\right)\left(\sum_{cyc} \frac{1}{a}\right) = (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) =$$

$$= 3 + \frac{a+c}{b} + \frac{b+a}{c} + \frac{c+b}{a} =$$

$$= 3 + \frac{ac(a+c) + ba(b+a) + cb(c+b)}{abc} =$$

$$= 1 + \frac{abc + abc + ac(a+c) + ba(b+a) + cb(c+b)}{abc} =$$

$$= 1 + \frac{ab(a+b+c) + ac(a+b+c) + bc(b+c)}{abc} =$$

$$= 1 + \frac{a(a+b+c)(b+c) + bc(b+c)}{abc} =$$

$$= 1 + \frac{(b+c)(a^2 + ab + ac + bc)}{abc} =$$

$$= 1 + \frac{(a+b)(b+c)(c+a)}{abc} \overset{\text{CESARO}}{\geq}$$

$$\geq 1 + \frac{8abc}{abc} = 1 + 8 = 9$$

Equality holds for $a = b = c$. $\qquad\qquad\square$

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