PP46722

MIHÁLY BENCZE - ROMANIA

If $a, b > 0$ then:

$$\frac{a^3+b^3}{ab}+a^2+b^2+\frac{1}{a}+\frac{1}{b}+\frac{8}{a+b}\geq 10$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\frac{a^3+b^3}{ab}+a^2+b^2+\frac{1}{a}+\frac{1}{b}+\frac{8}{a+b}=$$

$$=\frac{a^2}{b}+\frac{b^2}{a}+\frac{a^2}{1}+\frac{b^2}{1}+\frac{1^2}{a}+\frac{1^2}{b}+\frac{8}{a+b}\geq$$

$$\overset{\text{BERGSTRÖM}}{\geq}\frac{(a+b)^2}{a+b}+\frac{(a+b)^2}{1+1}+\frac{(1+1)^2}{a+b}+\frac{8}{a+b}=$$

$$=(a+b)+\frac{(a+b)^2}{2}+\frac{12}{a+b}=$$

$$=(a+b)+\frac{4}{a+b}+\frac{(a+b)^2}{2}+\frac{4}{a+b}+\frac{4}{a+b}\geq$$

$$\overset{\text{AM-GM}}{\geq}2\cdot\sqrt{(a+b)\cdot\frac{4}{a+b}}+3\sqrt[3]{\frac{(a+b)^2}{2}\cdot\frac{4}{a+b}\cdot\frac{4}{a+b}}=$$

$$=2\cdot 2+3\cdot 2=10$$

Equality holds for $a=b=1$. □

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