

PP46813

MIHÁLY BENCZE - ROMANIA

If $a, b, c \in (0, 1]$ then:

$$\sum_{cyc} \frac{b}{a(b+1)^2} \geq \frac{3}{4}$$

Solution by Daniel Sitaru.

$$(1) \quad a \leq 1 \Rightarrow a + 1 \leq 2 \Rightarrow \frac{1}{a+1} \geq \frac{1}{2}$$

$$(2) \quad b \leq 1 \Rightarrow b + 1 \leq 2 \Rightarrow \frac{1}{b+1} \geq \frac{1}{2}$$

$$(3) \quad c \leq 1 \Rightarrow c + 1 \leq 2 \Rightarrow \frac{1}{c+1} \geq \frac{1}{2}$$

By (1); (2); (3):

$$\frac{1}{(a+1)(b+1)(c+1)} \geq \frac{1}{8}$$

$$(4) \quad \frac{1}{(a+1)^2(b+1)^2(c+1)^2} \geq \frac{1}{64}$$

$$\begin{aligned} \sum_{cyc} \frac{b}{a(b+1)^2} &\stackrel{\text{AM-GM}}{\geq} 3 \cdot \sqrt[3]{\frac{abc}{abc(a+1)^2(b+1)^2(c+1)^2}} = \\ &= 3 \cdot \sqrt[3]{\frac{1}{(a+1)^2(b+1)^2(c+1)^2}} \stackrel{(4)}{\geq} 3 \cdot \sqrt[3]{\frac{1}{64}} = \frac{3}{4} \end{aligned}$$

Equality holds for $a = b = c$. □

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