PP46816

MIHÁLY BENCZE - ROMANIA

Prove that:

$$\frac{\pi}{4}+\int_1^e \frac{\arctan x}{x}dx > \arctan e$$

Solution by Daniel Sitaru.

Let be $f:(0,\infty)\to\mathbb{R}; f(x)=\arctan x-\frac{x}{1+x^2}$

$$f'(x)=\frac{2x^2}{(1+x^2)^2}>0\Rightarrow f \text{ increasing}$$

$$\inf_{x>0} f(x)=\lim_{\substack{x\to 0\\ x>0}} f(x)=0\Rightarrow f(x)>0; (\forall)x>0$$

$$\Rightarrow \arctan x-\frac{x}{1+x^2}>0\Rightarrow \arctan x>\frac{x}{1+x^2}$$

$$\Rightarrow \frac{\arctan x}{x}>\frac{1}{1+x^2}$$

$$\int_1^e \frac{\arctan x}{x}dx>\int_1^e \frac{1}{1+x^2}dx=\arctan e-\arctan 1$$

$$\int_1^e \frac{\arctan x}{x}dx>\arctan e-\frac{\pi}{4}$$

$$\frac{\pi}{4}+\int_1^e \frac{\arctan x}{x}dx>\arctan e$$

□

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