PP46893

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\left(\frac{a}{\sin\frac{A}{2}}\right)^2 = 8R(4R+r)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc}\left(\frac{a}{\sin\frac{A}{2}}\right)^2 = \sum_{cyc}\frac{a^2}{\sin^2\frac{A}{2}} = \sum_{cyc}\frac{a^2}{\frac{(s-b)(s-c)}{bc}} =$$

$$= abc\sum_{cyc}\frac{a}{(s-b)(s-c)} = \frac{abc}{(s-a)(s-b)(s-c)}\sum_{cyc}a(s-a) =$$

$$= \frac{abcs}{s(s-a)(s-b)(s-c)}\left(s\sum_{cyc}a - \sum_{cyc}a^2\right) =$$

$$= \frac{abcs}{F^2}\cdot(s\cdot 2s - 2s^2 + 2r^2 + 8Rr) =$$

$$= \frac{4RFs}{F^2}\cdot 2r(r+4R) =$$

$$= \frac{4Rs}{F}\cdot 2r(4R+r) =$$

$$= \frac{8RF}{F}\cdot(4R+r) = 8R(4R+r)$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com