

PP46894

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{a^3}{\sin^2 \frac{A}{2}} = 16Rs(R+r)$$

Solution by Daniel Sitaru, Claudia Nănuți.

$$\begin{aligned} \sum_{cyc} \frac{a^3}{\sin^2 \frac{A}{2}} &= \sum_{cyc} \frac{a^3}{\frac{(s-b)(s-c)}{bc}} = \\ &= abc \sum_{cyc} \frac{a^2}{(s-b)(s-c)} = \frac{abc}{(s-a)(s-b)(s-c)} \sum_{cyc} a^2(s-a) = \\ &= \frac{abcs}{s(s-a)(s-b)(s-c)} \cdot 4rs(R+r) = \\ &= \frac{abcs^2}{F^2} \cdot 4r(R+r) = \\ &= \frac{abcs^2}{r^2s^2} \cdot 4r(R+r) = \\ &= \frac{abc}{r} \cdot 4(R+r) = \frac{4Rrs}{r} \cdot 4(R+r) = \\ &= 16Rs(R+r) \end{aligned}$$

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