

PP46900

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{a}{\sin^2 \frac{A}{2}} = \frac{4sR}{r}$$

Solution by Daniel Sitaru, Claudia Nănuți.

$$\begin{aligned} \sum_{cyc} \frac{a}{\sin^2 \frac{A}{2}} &= \sum_{cyc} \frac{a}{\frac{(s-b)(s-c)}{bc}} = \\ &= abc \sum_{cyc} \frac{1}{(s-b)(s-c)} = \frac{abc}{(s-a)(s-b)(s-c)} \sum_{cyc} (s-a) = \\ &= \frac{abcs}{s(s-a)(s-b)(s-c)} \left(3s - \sum_{cyc} a \right) = \\ &= \frac{abcs}{F^2} (3s - 2s) = \frac{abcs^2}{r^2 s^2} = \\ &= \frac{abc}{r^2} = \frac{4Rrs}{r^2} = \frac{4Rs}{r} \end{aligned}$$

□

MATHEMATICS DEPARTMENT, NATIONAL ECONOMIC COLLEGE "THEODOR COSTESCU", DROBETA
TURNU - SEVERIN, ROMANIA

Email address: dansitaru63@yahoo.com